Prove the reduction formula $\int \sec^n u\, du = \frac{1}{n-1}\sec^{n-2} u \tan u + \frac{n-2}{n-1}\int \sec^{n-2} u\, du$.

SCORE: _____ / 6 PTS

<u>NOTE: You must show how to get this formula.</u>

<u>You will receive 0 credit if your "proof" is differentiating both sides of the equation.</u>

u	dv
$\sec^{n-2} u$	$\sec^2 u$
$(n-2)\sec^{n-2} u \tan u$	$\tan u$

$$\int \sec^n u\, du = \sec^{n-2} u \tan u - (n-2)\int \sec^{n-2} u \tan^2 u\, du \quad 2$$

$$= \sec^{n-2} u \tan u - (n-2)\int \sec^{n-2} u (\sec^2 u - 1)\, du \quad 3$$

$$= \sec^{n-2} u \tan u - (n-2)\int \sec^n u\, du + (n-2)\int \sec^{n-2} u\, du$$

$$(n-1)\int \sec^n u\, du = \sec^{n-2} u \tan u + (n-2)\int \sec^{n-2} u\, du \quad 1$$

$$\int \sec^n u\, du = \frac{1}{n-1}\sec^{n-2} u \tan u + \frac{n-2}{n-1}\int \sec^{n-2} u\, du$$

Evaluate $\int \frac{(\ln x)^2}{x^2}\,dx$.

SCORE: _____ / 4 PTS

u		dv
$(\ln x)^2$	$+$	x^{-2}
$\frac{2\ln x}{x}$		$-x^{-1}$
$2\ln x$		$-x^{-2}$
	$-$	
$\frac{2}{x}$		x^{-1}
2	$+$	x^{-2}
0		$-x^{-1}$

$$\underbrace{-x^{-1}(\ln x)^2}_{1}\ \underbrace{-2x^{-1}\ln x}_{1\frac{1}{2}}\ \underbrace{-2x^{-1}}_{1\frac{1}{2}} + C$$

-1 IF YOU FORGOT "$+C$"

Evaluate $\int \sin^2 x \cos^5 x \, dx$ IF YOU USED THE REDUCTION FORMULA, YOU MAY EARN FULL CREDIT IF YOU WRITE AN ALGEBRAIC PROOF THAT YOUR FINAL ANSWER IS EQUIVALENT

SCORE: _____ / 4 PTS

$u = \sin x \longrightarrow du = \cos x \, dx$

1

1 $\int u^2(1-u^2)^2 \, du = \int (u^2 - 2u^4 + u^6) \, du$ $\frac{1}{2}$

$\frac{1}{2}$ $= \frac{1}{3}u^3 - \frac{2}{5}u^5 + \frac{1}{7}u^7 + C$

$= \frac{1}{3}\sin^3 x - \frac{2}{5}\sin^5 x + \frac{1}{7}\sin^7 x + C$

1

-1 IF YOU FORGOT "+C"

Evaluate $\int e^{-2x} \cos 4x \, dx$. **SCORE: _____ / 5 PTS**

$$\begin{array}{cc} \underline{u} & \underline{dv} \\ \cos 4x & +\ e^{-2x} \\ -4\sin 4x & -\ -\frac{1}{2}e^{-2x} \\ -16\cos 4x & +\ \frac{1}{4}e^{-2x} \end{array}$$

$$\int e^{-2x}\cos 4x \, dx = \underbrace{-\tfrac{1}{2}e^{-2x}\cos 4x}_{} + \underbrace{e^{-2x}\sin 4x}_{} - \underbrace{\int 4e^{-2x}\cos 4x \, dx}_{}$$

1 1 1

$$\underbrace{5\int e^{-2x}\cos 4x \, dx = -\tfrac{1}{2}e^{-2x}\cos 4x + e^{-2x}\sin 4x}_{}$$

1

$$\int e^{-2x}\cos 4x \, dx = \underbrace{-\tfrac{1}{10}e^{-2x}\cos 4x + \tfrac{1}{5}e^{-2x}\sin 4x}_{} + C$$

1

-1 IF YOU FORGOT "+C"

Evaluate $\int \cos^{-1} x \, dx$. **SCORE: _____ / 4 PTS**

u	dv
$\cos^{-1} x$	1
$-\frac{1}{\sqrt{1-x^2}}$	x
-1	$\frac{x}{\sqrt{1-x^2}}$
0	$-\sqrt{1-x^2}$

$$\underbrace{x\cos^{-1}x}_{1} + \underbrace{\int \frac{x}{\sqrt{1-x^2}}\,dx}_{1} \leftarrow \quad u = 1-x^2, \quad du = -2x\,dx$$

$$\underbrace{\int -\frac{1}{2}\frac{1}{\sqrt{u}}\,du}_{1} = -\sqrt{u} = -\sqrt{1-x^2}$$

OR

$$= x\cos^{-1}x \underbrace{- \sqrt{1-x^2}}_{1} + C$$

-1 IF YOU FORGOT "$+C$"

Evaluate $\int (x^2-4)^{\frac{3}{2}}\,dx$.

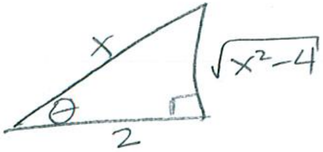
SCORE: _____ / 7 PTS

$$\int 4^{\frac{3}{2}}\left(\frac{x^2}{4}-1\right)^{\frac{3}{2}}dx$$

$\frac{x^2}{4} = \sec^2\theta$

1 $x = 2\sec\theta$

$dx = 2\sec\theta\tan\theta\,d\theta$



$$= \int 8(\sec^2\theta-1)^{\frac{3}{2}}\cdot 2\sec\theta\tan\theta\,d\theta$$

$$= 16\int \tan^3\theta\cdot\sec\theta\tan\theta\,d\theta$$

$$= 16\int \sec\theta\tan^4\theta\,d\theta$$

$$= 16\int \sec\theta(\sec^2\theta-1)^2\,d\theta$$

$$= 16\int (\sec^5\theta - 2\sec^3\theta + \sec\theta)\,d\theta$$

$$= 16\left[\frac{1}{4}\sec^3\theta\tan\theta + \frac{3}{4}\int\sec^3\theta\,d\theta - 2\int\sec^3\theta\,d\theta + \int\sec\theta\,d\theta\right]$$

$\frac{1}{2}$ POINT EXCEPT AS NOTED

-1 IF YOU FORGOT "+C"

$$= 4\sec^3\theta\tan\theta - 20\int \sec^3\theta\, d\theta + 16\int \sec\theta\, d\theta$$

$$= 4\sec^3\theta\tan\theta - 20\left(\tfrac{1}{2}(\sec\theta\tan\theta + \ln|\sec\theta + \tan\theta|)\right) + 16\ln|\sec\theta + \tan\theta| + C$$

$$= 4\sec^3\theta\tan\theta - 10\sec\theta\tan\theta + 6\ln|\sec\theta + \tan\theta| + C$$

$$= 4\left(\frac{x}{2}\right)^3 \frac{\sqrt{x^2-4}}{2} - 10\left(\frac{x}{2}\right)\frac{\sqrt{x^2-4}}{2} + 6\ln\left|\frac{x}{2} + \frac{\sqrt{x^2-4}}{2}\right| + C$$

$$= \frac{1}{4}x^3\sqrt{x^2-4} - \frac{5}{2}x\sqrt{x^2-4} + 6\ln\left|x + \sqrt{x^2-4}\right| + C$$